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INTERACTION OF HYDRODYNAMIC EXTERNAL DISTURBANCES WITH THE BOUNDARY LAYER

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Disturbances produced by external flow vorticity in a supersonic boundary layer are studied. It is shown that both vortical and nonvortical waves play an important role. The calculations are performed for subsonic and supersonic flows for a Mach number $\mathrm{M} = 2$.

Introduction. At present, in solving problems of turbulence arising in boundary layers, the study of their receptivity to external disturbances acquires great significance. In this case, disturbance generation in boundary layers is attributed to stationary and nonstationary factors (surface vibration, acoustic waves, external turbulence, etc.). One of the first authors to treat this problem and to introduce the term "receptivity" was Morkovin [1]. There are a number of experimental and theoretical studies of subsonic boundary-layer receptivity. A detailed survey can be found in [2, 3]. Much less is known, however, about the receptivity of a supersonic boundary layer, with all the publications in this field being devoted to studies of interaction of acoustic waves with the boundary layer [4–6]. At the same time, a supersonic flow may contain not only acoustic disturbances, but also vortical and entropy or heat waves. The interaction of those with a supersonic boundary layer has not yet been adequately investigated. The problem of receptivity of vortical disturbances at subsonic velocities has been analyzed in some publications, but there is no agreement among the authors about the mechanism of disturbance generation in the boundary layer. In this work, excitation of stationary and nonstationary waves by external hydrodynamic disturbances in subsonic and supersonic boundary layers was calculated.

Formulation of the Problem and Governing Equations. A linear statement of the problem is considered. A compressible gas flow in the boundary layer on a flat plate is taken as the initial undisturbed flow.

Following Petrov [7], the disturbances in the boundary layer are considered in the orthogonal system of coordinates (ξ, ψ, z) fitted to the stream surfaces of the main flow. Here ψ is the stream function, $\xi = x + O(\mathrm{Re}^{-2})$ for the plate, $\mathrm{Re} = \sqrt{u_\infty x/\nu_\infty}$ is the Reynolds number, u_∞ and ν_∞ are the velocity and kinematic viscosity of the incident flow, and x, y, and z are the longitudinal, normal-to-wall, and transversal coordinates of the Cartesian coordinate system with the origin on the plate edge. The gas is perfect with a constant Prandtl number Pr. Making use of the estimates with respect to the integer powers of the Reynolds number Re as in [8] and rejecting terms of order Re^{-2} with respect to the dominant terms of the linearized Navier–Stokes equations [7] for disturbances of the form $\tilde{a}(\xi, \psi) \exp(i\alpha\xi + i\beta z - i\omega t)$, we obtain

$$\begin{gathered}
\partial_2 \tilde{v} = -(\partial_2 \ln \rho)\tilde{v} - [i\alpha - (\partial_1 \ln u) + \partial_1]\tilde{u} - i\beta\tilde{w} - u_c\tilde{\rho}/\rho - g_m u \partial_1 \tilde{p} + u\partial_1(\tilde{T}/T), \\
\partial_2[\tilde{p} + 2\mu(i\alpha\tilde{u} + i\beta\tilde{w} - 2\tilde{e}_0/3)] = -\rho(h_1 u + d_t)\tilde{v} + i\alpha\tilde{\tau}_{12} + i\beta\tilde{\tau}_{23}, \\
\partial_2\tilde{\tau}_{12} = (i\alpha + \partial_1)\tilde{p} + \rho(\partial_2 u)\tilde{v} + \rho(\partial_1 u + d_t)\tilde{u} + u(\partial_1 u)\tilde{\rho} - i\alpha\tilde{\tau}_{11} - i\beta\tilde{\tau}_{13}, \\
\partial_2\tilde{u} = -(i\alpha + \partial_1)\tilde{v} - (\partial_2 u)\tilde{\mu}/\mu + \tilde{\tau}_{12}/\mu, \\
\partial_2\tilde{\tau}_{23} = i\beta\tilde{p} + \rho d_t\tilde{w} - i\alpha\tilde{\tau}_{13} - i\beta\tilde{\tau}_{33}, \qquad \partial_2\tilde{w} = -i\beta\tilde{v} + \tilde{\tau}_{23}/\mu,
\end{gathered} \tag{1}$$

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$$\partial_2\tilde{q} = i\omega\tilde{p} + [\rho(\partial_2 H) - i\alpha\mu(\partial_2 u)]\tilde{v} + (\partial_1 H)(\rho\tilde{u} + u\tilde{\rho}) + (\alpha^2+\beta^2)\mu\tilde{h}/\mathrm{Pr} + \rho d_t\tilde{H} - u(i\alpha\tilde{\tau}_{11} + i\beta\tilde{\tau}_{13}),$$

$$\partial_2\tilde{h} = -\mathrm{Pr}\,(\partial_2 u)\tilde{u} - (\partial_2 h)\tilde{\mu}/\mu + \mathrm{Pr}\,(\tilde{q} - u\tilde{\tau}_{12})/\mu,$$

where $\partial_1 = \partial/\partial\xi$, $\partial_2 = \rho u\partial/\partial\psi$, $d_t = u_c + u\partial_1$, $u_c = i\alpha u - i\omega$, $h_1 = -\partial_1 \ln(\rho u)$, $\tilde{\tau}_{11} = 2\mu(i\alpha\tilde{u} - \tilde{e}_0/3)$, $\tilde{\tau}_{13} = \mu(i\alpha\tilde{w} + i\beta\tilde{u})$, $\tilde{\tau}_{33} = 2\mu(i\beta\tilde{w} - \tilde{e}_0/3)$, $\tilde{e}_0 = -(\partial_2 \ln\rho)\tilde{v} - u_c\tilde{\rho}/\rho$, u is the velocity, T is the temperature, ρ is the density, $H = h + u^2/2$ is the total enthalpy, μ is the viscosity, $\tilde{p}$ is the pressure, $\tilde{v}$ and $\tilde{w}$ are the complex amplitudes of the transversal and normal-to-stream-surface components of velocity disturbance, $\tilde{\rho}/\rho = g_m\tilde{p} - \tilde{T}/T$ ($\tilde{T} = g_{m1}\tilde{h}$), $\tilde{H} = \tilde{h} + u\tilde{u}$, $g_m = 1/p$, and $g_{m1} = 1/c_p$ (c_p is the specific heat capacity of the gas at constant pressure). The form of the equations is not changed by normalization to the following scales: ν_∞/u_∞ for length, ν_∞/u_∞^2 for time, μ_∞ for viscosity and the stream function, u_∞ for velocity and its disturbances, T_∞ for temperature, ρ_∞ for density, u_∞^2 for enthalpy, $\rho_\infty u_\infty^2$ for pressure and disturbances of viscous stresses, $\rho_\infty u_\infty^3$ for the quantity $\tilde{q}$, and u_∞^2/T_∞ for specific heat capacity (the subscript ∞ refers to the values in the incident flow). In this case, we have $g_m = \gamma\mathrm{M}^2$ and $g_{m1} = (\gamma-1)\mathrm{M}^2$, where $\gamma = c_p/c_V$ is the ratio of specific heats, and M is the Mach number.

We substitute the independent variables: $\mathrm{Re} = \sqrt{\xi}$, $d\eta = df/u$, and $f = \psi/\mathrm{Re}$. Then, we obtain $\partial_1\tilde{a} = (1/\mathrm{Re})(\partial\tilde{a} + f_1\tilde{a}')$ and $\partial_2\tilde{a} = \rho\tilde{a}'/\mathrm{Re}$, where $\partial = 0.5\partial/\partial\mathrm{Re}$; the prime stands for the derivative with respect to η, and $f_1 = -f/(2\mathrm{Re}u)$. In this case, the equations based on the estimates of the critical layer have the following form:

$$\tilde{v}' = -g_m uT\partial\tilde{p} + \rho T'\tilde{v} - T(f_0 u' + \partial)\tilde{u} - \tilde{u}_w - i_c T\tilde{r} - (f_2\rho T' - u\partial)\tilde{T} - f_1 T\tilde{u}' + f_2\tilde{T}',$$

$$\tilde{p}' = -(i_c + r_h u)\tilde{v} + i_x\tilde{\tau}_{12} + i_z\tilde{\tau}_{23} - 2\mu_r\tilde{u}_w',$$

$$\tilde{\tau}_{12}' = (i_x + T\partial)\tilde{p} + \rho u'\tilde{v} + (i_c + f_1 u' + u\partial)\tilde{u} + f_2 u'\tilde{r} - \tilde{i}_t + f_2\tilde{u}',$$

$$\tilde{u}' = -i_x\tilde{v} - u'\mu_t\tilde{T} + \tilde{\tau}_{12}/\mu_r, \tag{2}$$

$$\tilde{\tau}_{23}' = i_z\tilde{p} + (i_c - \mu_a + u\partial)\tilde{w} - i_z\mu_r\tilde{u}_w + f_2\tilde{w}', \qquad \tilde{w}' = -i_z\tilde{v} + \tilde{\tau}_{23}/\mu_r,$$

$$\tilde{q}' = i\omega\mathrm{Re}T\tilde{p} + \rho H'\tilde{v} + (i_c u + f_1 H' + f_2 u' + u^2\partial)\tilde{u} + (i_c - \mu_a/\mathrm{Pr} + u\partial)\tilde{h} + f_2 H'\tilde{r} - u\tilde{i}_t + f_2 u\tilde{u}' + f_2\tilde{h}',$$

$$\tilde{h}' = -\mathrm{Pr}\,u'\tilde{u} - h'\mu_t\tilde{T} + \mathrm{Pr}\,(\tilde{q} - u\tilde{\tau}_{12})/\mu_r.$$

Here $\tilde{u}_w = i_x\tilde{u} + i_z\tilde{w}$, $\tilde{i}_t = i_x\mu_r\tilde{u}_w + \mu_a\tilde{u}$, $\mu_a = (i_x^2 + i_z^2)\mu_r$, $\tilde{p} = \tilde{\pi} - 2\mu(i\alpha\tilde{u} + i\beta\tilde{w} - 2\tilde{e}_0/3)$, $\tilde{r} = \tilde{\rho}/\rho = g_m\tilde{p} - \rho\tilde{T}$, $i_c = \mathrm{Re}u_c = i\mathrm{Re}(u\alpha - \omega)$, $i_x = i\alpha\mathrm{Re}T$, $i_z = i\beta\mathrm{Re}T$, $r_h = \mathrm{Re}h_1 = f_0 u' + f_1\rho T'$, $f_0 = -f_1/u$, $f_2 = f_1 u$, $\mu_r = \mu\rho/\mathrm{Re}$, and $\mu_t = d\ln\mu/dT$.

With allowance for the substitutions, systems (1) and (2) have the form

$$\boldsymbol{Z}' = (A + D\partial)\boldsymbol{Z}, \tag{3}$$

where $\boldsymbol{Z} = (\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}, \tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q})$ in system (2) and A and D are the square matrices of the specified functions Re and η.

The parabolized system (3) is solved under the following boundary conditions. The disturbances of velocities and temperature (or heat flux) on the surface are zero:

$$\tilde{v}(0) = \tilde{u}(0) = \tilde{w}(0) = \tilde{T}(0) = 0 \qquad [\tilde{T}'(0) = 0]. \tag{4}$$

Outside the boundary layer, the disturbances are obtained from their corresponding free-stream values (without the model).

Numerical Scheme and Results. On using the approximation $\partial\tilde{a}/\partial\mathrm{Re} \approx (\tilde{a} - \tilde{a}_0)/\Delta\mathrm{Re}$ ($\Delta\mathrm{Re} = \mathrm{Re} - \mathrm{Re}_0$ is the step of the marching scheme and the subscript 0 corresponds to the values calculated at the previous step), system (3) is transformed into the system of ordinary differential equations

$$\boldsymbol{Z}' = A\boldsymbol{Z} + B(\boldsymbol{Z} - \boldsymbol{Z}_0). \tag{5}$$

Its general solution is a superposition of four linearly independent solutions of a homogeneous system and an arbitrary quotient: $\boldsymbol{Z} = \sum_{m=1}^{4} C_m(x)\boldsymbol{Z}_m + \boldsymbol{Z}_5$. Herewith, the coefficients C_m are found from the conditions on the surface (4). The selection of four fundamental solutions should be made from those decaying as $\eta \to \infty$. The quotient should be a solution that describes external (undisturbed by the boundary layer) waves. It is, therefore, assumed that $\boldsymbol{Z}_5$ is known. For external acoustic waves, where $\alpha \neq \omega$ and $\alpha = O(\beta)$, four fundamental solutions in [8] were based on the theory of parallel flows:

$$\boldsymbol{Z}_1 = \begin{bmatrix} 0 \\ -i\alpha \\ \lambda_1 \\ 0 \\ 0 \\ \lambda_1^2 + \alpha^2 \\ \alpha\beta \\ \lambda_1^2 + \alpha^2 \end{bmatrix}, \qquad \boldsymbol{Z}_2 = \begin{bmatrix} 0 \\ -i\beta \\ 0 \\ \lambda_2 \\ 0 \\ \alpha\beta \\ \lambda_2^2 + \beta^2 \\ \alpha\beta \end{bmatrix},$$

$$\boldsymbol{Z}_3 = \begin{bmatrix} B - B^2/\mathrm{Pr} \\ \lambda_3 B_1 \\ i\alpha B_1 \\ i\beta B_1 \\ B \\ 2i\alpha\lambda_3 B_1 \\ 2i\beta\lambda_3 B_1 \\ \lambda_3 B/\mathrm{Pr} + 2i\alpha\lambda_3 B_1 \end{bmatrix}, \qquad \boldsymbol{Z}_4 = \begin{bmatrix} A - A^2/\mathrm{Pr} \\ \lambda_4 A_1 \\ i\alpha A_1 \\ i\beta A_1 \\ A \\ 2i\alpha\lambda_4 A_1 \\ 2i\beta\lambda_4 A_1 \\ \lambda_4 A/\mathrm{Pr} + 2i\alpha\lambda_4 A_1 \end{bmatrix},$$

$$\boldsymbol{Z} = (\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}, \tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q}),$$

$$\lambda_1 = \lambda_2 = \sqrt{i(\alpha - \omega) + \lambda^2}, \qquad \lambda_3 = \sqrt{Bi(\alpha - \omega) + \lambda^2},$$

$$\lambda_4 = \sqrt{Ai(\alpha - \omega) + \lambda^2}, \qquad \lambda = \sqrt{\alpha^2 + \beta^2},$$

$$C = \mathrm{Pr} + \frac{g_m i(\alpha - \omega) - (4/3) g_{m1} i(\alpha - \omega)\mathrm{Pr}}{1 + (4/3) g_m i(\alpha - \omega)},$$

$$A = \frac{\mathrm{Pr}\,(g_m - g_{m1}) i(\alpha - \omega)}{(1 + (4/3) i(\alpha - \omega))C}\Big(1 + \frac{\mathrm{Pr}\,(g_m - g_{m1}) i(\alpha - \omega)}{(1 + (4/3) i(\alpha - \omega))C^2}\Big),$$

$$B = C - A, \qquad A_1 = g_{m1} - g_m(1 - A/\mathrm{Pr}\,), \qquad B_1 = g_{m1} - g_m(1 - B/\mathrm{Pr}\,).$$

In the present paper, the case is considered where the external disturbances are entrained by the flow. For a parallel flow and negligibly small viscosity of the gas, we have $\omega = \alpha$; therefore, $u_c = 0$ outside the boundary layer. In this case, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -\lambda$, and three out of four constructed fundamental vectors $\boldsymbol{Z}_m$ are linearly independent, e.g., $\boldsymbol{Z}_1$, $\boldsymbol{Z}_3$, and $\boldsymbol{Z}_4$. It should be noted that $\tilde{p} = 0$ for all those vectors. At the same time, it can be shown that the pressure disturbance in a homogeneous flow satisfies the Laplace equation $\Delta\tilde{p} = 0$. Therefore, alongside the solutions corresponding to $\tilde{p} = 0$, a solution satisfying the Laplace equation can be constructed for $\tilde{p} = \exp(-\lambda y)$. Therewith, the velocity disturbances $\boldsymbol{V} = (\tilde{u}, \tilde{v}, \tilde{w})$ satisfy the equation

$$\mathrm{grad}\,\tilde{p} = \Big(\frac{d^2}{dy^2} - \lambda^2\Big)\boldsymbol{V}.$$

By virtue of the fact that the exponent of $\tilde{p}$ is $-\lambda$, the solution has the form $\boldsymbol{V} = (\boldsymbol{a} + \boldsymbol{b}y)\exp(-\lambda y)$. Leaving out all the intermediate calculations, we show one of the possible vectors on the external edge of the boundary layer: $\boldsymbol{Z}_2 = (2\lambda, 1, 0, 0, 0, 0, 0, 0)$. We have, therefore, four linearly independent solutions on the edge of the boundary layer, continuously transforming into the decaying ones as $y \to \infty$.

In addition, the fifth vector $\boldsymbol{Z}_5$ corresponding to external disturbances should be specified at the boundary-layer edge. It can be a linear combination of four linearly independent solutions increasing with respect to y and analogous to $\boldsymbol{Z}_m$, where $-\lambda_m$ are substituted for λ_m. The calculations were mostly performed for the case where $\boldsymbol{Z}_5$ was a vector of nonvortical disturbances obtained from $\boldsymbol{Z}_2$ by substituting $-\lambda_2$ for λ_2 and normalized to the fourth component. Under such normalization, the amplitude of the transversal velocity of external disturbances at the boundary-layer edge is equal to unity. As the calculation results show, the most intensive disturbances of the

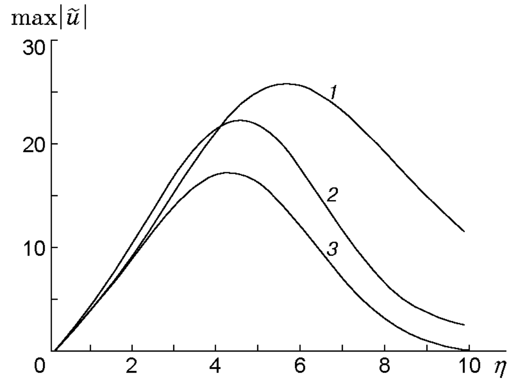

Fig. 1. Distributions of the amplitudes of longitudinal velocity disturbances over the boundary layer for three approximations: curve 1 refers to the parallel-flow model, curve 2 to the local model, and curve 3 to the model based on parabolized stability equations.

longitudinal velocity inside the boundary layer occur in the low-frequency range and provided that $\alpha \ll \beta$. Their maximum is observed for waves with a characteristic value of the wavenumber $\beta = \beta^*$.

Characteristic distributions of the amplitudes $|\tilde{u}|$ over the boundary layer under the specified vectors are shown in Fig. 1 for three approximations (parallel-flow model, local model, and model based on parabolized stability equations). The local model was developed by neglecting terms proportional to $\partial \boldsymbol{Z}$ in (3). The behavior of temperature-disturbance amplitudes is similar. It is clear that, in the first of the considered approximations, the value of $|\tilde{u}|$ at the boundary-layer edge is of the same order of magnitude as the maximum value inside the boundary layer. From calculations using the local model, it follows that the disturbances $|\tilde{u}|$ at the boundary-layer edge are close to zero. Making use of more exact parabolized equations of stability leads to a further decrease in $|\tilde{u}|$ at the boundary-layer edge. Nonzero values of $|\tilde{u}|$ at the boundary-layer edge for the parallel-flow and local models are due to the employed vector $\boldsymbol{Z}_1$, in which $\tilde{u} \neq 0$. This also holds for the distribution of temperature disturbances, which do not tend to zero at the boundary-layer edge because the vector $\boldsymbol{Z}_3$ with $\tilde{h} \neq 0$ is used. Thus, the solution obtained using the vectors $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_3$ is inconsistent with the solution based on the local model and parabolized equations.

Further, other vectors $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_3$ are presented, obtained from the exact equations of motion, continuity, and heat conduction for a homogeneous flow outside the boundary layer. It should be noted that $\tilde{p} = 0$ and $\tilde{w} = 0$ for the vectors $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_3$. In addition, $\tilde{h} = 0$ for the vector $\boldsymbol{Z}_1$ and $\tilde{u} = 0$ for the vector $\boldsymbol{Z}_3$.

For $\tilde{p} = \tilde{T} = \tilde{w} = 0$, we have two equations

$$\frac{\partial \tilde{u}}{\partial x} = \frac{\partial^2 \tilde{u}}{\partial y^2} - \lambda^2 \tilde{u}^2, \qquad \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + i\alpha \tilde{u} = 0.$$

Henceforward, the wavenumbers and coordinates are normalized to the length scale ν_∞/u_∞. The first of those equations has a fundamental solution

$$\tilde{u} = (1/\sqrt{x}) \exp(-\lambda^2 x - y^2/(4x)).$$

Therefore, for $y \gg 1$, we have $\tilde{u} \ll \partial \tilde{u}/\partial y \ll \partial^2 \tilde{u}/\partial y^2$, and instead of the second equation the following may be taken:

$$\frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{\partial \tilde{v}}{\partial y} = \frac{\partial}{\partial y}\Big(\frac{\partial \tilde{u}}{\partial y} + \tilde{v}\Big) \approx 0.$$

Thus, $\tilde{v} = -\partial \tilde{u}/\partial y$. By virtue of the fact that $\tilde{\tau}_{12} = \partial \tilde{u}/\partial y + i\alpha \tilde{v}$, $\tilde{\tau}_{23} = \partial \tilde{w}/\partial y + i\beta \tilde{v}$, and $\tilde{q} = \tilde{\tau}_{12} + \lambda \partial T/\partial y$ for $\tilde{T} = \tilde{w} = \tilde{p} = 0$ and provided that $\partial \tilde{v}/\partial y \ll \tilde{v}$ and $\tilde{u} \ll \partial \tilde{u}/\partial y$, we have $\boldsymbol{Z}_1 = (0, -1, 0, 0, 0, 1 - i\alpha, -i\beta, 1)$. For $\tilde{p} = \tilde{w} = \tilde{u} = 0$, the equations of heat conduction and continuity are used in the following form:

$$\frac{\partial \tilde{T}}{\partial x} = \frac{1}{\mathrm{Pr}}\Big(\frac{\partial^2 T}{\partial y^2} - \lambda^2 \tilde{T}\Big), \qquad -\frac{\partial \tilde{T}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0.$$

It follows from the above that we may take $\boldsymbol{Z}_3 = (0, -1, 0, 0, 0, -i\alpha, -i\beta, -1/g_{m1})$ as the third vector.

Figure 2 shows the amplitude distributions of longitudinal velocity disturbances over the boundary layer for various values of the frequency parameter at $\beta = 10^{-3}$. Figure 2 also shows a dependence proportional to $\eta u'$,

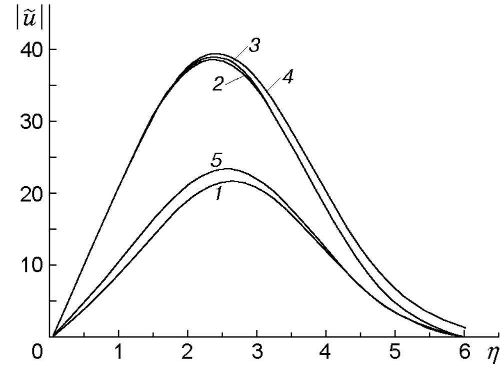

Fig. 2. Distributions of the amplitudes of longitudinal velocity disturbances over the boundary layer for $\beta = 10^{-3}$ and $\omega = 10^{-5}$ and M $= 0$ (1), $\omega = 10^{-6}$ and M $= 0$ (2), $\omega = 10^{-7}$ and M $= 0$ (3), and $\omega = 10^{-6}$ and M $= 2$ (4); curve 5 is the dependence proportional to $\eta u'$ [9].

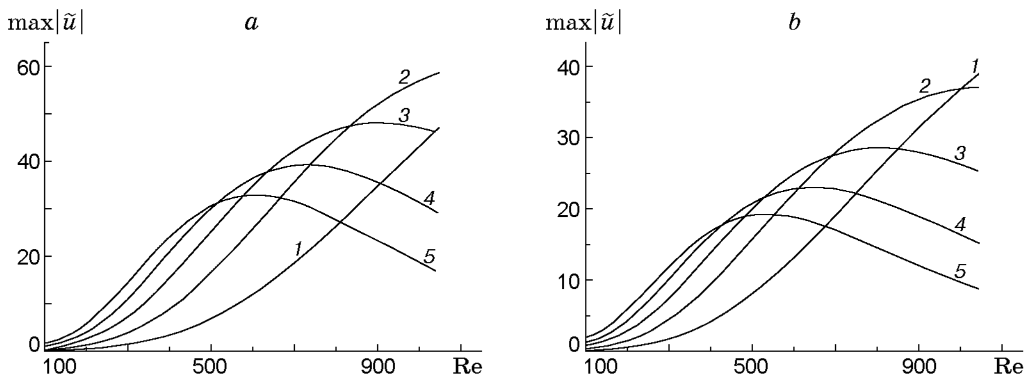

Fig. 3. Maximum values of the longitudinal velocity amplitude inside the boundary layer versus the Reynolds number for M $= 0$ (a) and M $= 2$ (b), $\omega = 10^{-6}$ and $\beta = 0.4 \cdot 10^{-3}$ (1), $0.6 \cdot 10^{-3}$ (2), $0.8 \cdot 10^{-3}$ (3), 10^{-3} (4), and $1.2 \cdot 10^{-3}$ (5).

obtained analytically in [9] for M $= 0$, which is a self-similar eigenfunction of the boundary layer equations for small values of β [10]. Similar results for M $= 0$ were obtained in [11, 12] by the method of optimum disturbances specified at $x = X_0$ in the absence of disturbances at the boundary-layer edge. A weak dependence of the profile of $|\tilde{u}|$ on changes in β and disturbances for $x = X_0$ is discussed in [12]. The independence of the amplitude profile $\tilde{u}$ on the parameters of disturbances at the external edge of the boundary layer seems to be caused by factors mentioned in [12]. The main of those factors is the fact that external disturbances generate eigen disturbances with an amplitude $\tilde{u} \sim yu'$.

Figure 3a shows the maximum values of the longitudinal velocity amplitude inside the boundary layer versus the Reynolds number for $\omega = 10^{-6}$, M $= 0$, and various wavenumbers β. It can be seen from Fig. 2 that the results coincide for $\omega = 10^{-6}$ and 10^{-7}. This implies that the disturbances may be considered stationary for $\omega \leqslant 10^{-6}$. It follows from the results shown in Fig. 3a that the maximum values of the amplitudes correspond to wavenumbers $(\beta\mathrm{Re})_1 = 0.7$ for fixed β and $(\beta\mathrm{Re})_2 = 0.55$ for a fixed Reynolds number (wavenumber based on the thickness $x/\mathrm{Re} = \mathrm{Re}$). These data are in good agreement with the results obtained by the method of optimum disturbances [11] excited in the region of the flat-plate leading edge. In addition, the value $(\beta\mathrm{Re})_2 = 0.55$ is in good agreement with the experimental results on evolution of streamwise structures [13]. It was found in [13] that the z period is approximately equal to the doubled thickness of the boundary layer, i.e., $\lambda \approx 2 \cdot 5x/\mathrm{Re}$, therefore, $\beta\mathrm{Re} = 2\pi/10 \approx 0.6$ (it is taken into account here that $x = \mathrm{Re}^2$).

The calculation results for M $= 2$ and $\omega = 10^{-6}$ are plotted in Fig. 3b. Qualitatively, they are close to the results obtained for M $= 0$. However, the amplitude of disturbances due to external hydrodynamic waves decreases if the boundary layer is supersonic.

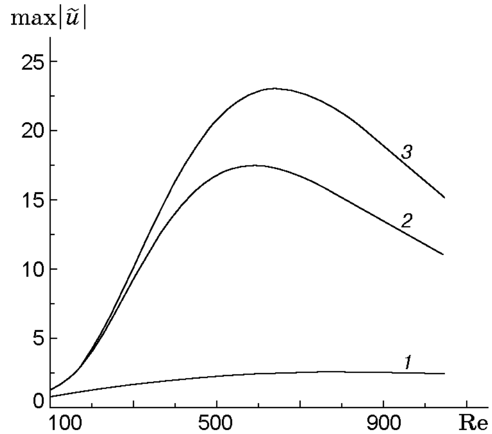

Fig. 4. Maximum values of the longitudinal velocity amplitude inside the boundary layer versus the Reynolds number for M $= 2$, $\beta = 10^{-3}$, and $\omega = 10^{-4}$ (1), 10^{-5} (2), and 10^{-6} (3).

Figure 4 shows the dependences $\max|\tilde{u}|(\mathrm{Re})$ for $\beta = 10^{-3}$. It follows from Fig. 4 that the amplitude significantly depends on the frequency parameter ω: as ω increases, the amplitude of disturbances decreases.

From the analysis of the phase velocity c versus Re (obtained from the phase growth along the x axis for the disturbance $\tilde{u}$ in the vicinity of its maximum, i.e., for $\eta \approx 2.3$) for $\omega = 0.3 \cdot 10^{-4}$, $\beta = 10^{-3}$ and M $= 0$ and 2, it follows that the phase velocity varies from 0.7 to 0.9 over the range of Re = 200–400. In experimental studies of propagation of incipient turbulence spots [3], it was found that their leading edge, which is in the range of high Re values, propagates with a velocity $c \approx 0.9$, and the trailing edge propagates with a velocity $c \approx 0.5$, which is in qualitative agreement with the results of our calculations. It should be taken into consideration that the phase velocity was calculated, whereas the object of calculations in experimental studies was most probably the group velocity. The present calculations show, however, that $\partial c/\partial\alpha \ll 1$, and the group velocity is close to the phase velocity.

As has been mentioned above, the results presented in this paper were obtained for nonvortical external disturbances. Additional calculations allowing for vortical disturbances in the external flow entrained by the main stream show that the character of disturbances inside the layer remains unchanged in this case. Only the relation between the amplitudes of external and internal disturbances undergoes some changes. The results obtained can be explained as follows. For high Reynolds numbers, the evolution of disturbances does not significantly depend on the presence of waves at the boundary-layer edge, and for low Reynolds numbers, the amplitude of disturbances inside the layer is determined by conditions at $x = X_0$. A change in external conditions at the boundary-layer edge is equivalent to a change in the initial data at $x = X_0$, which results in variation of the absolute value of the amplitude for $x > X_0$, whereas the character of the dependence of $|\tilde{u}|$ on η is determined by the first eigen-solution [12].

Conclusions. The following conclusions can be drawn from the present investigations.

1. External vortical and nonvortical waves entrained by the stream can excite high-intensity disturbances in the boundary layer.

2. The efficiency of disturbance generation increases with decreasing frequency, and its maximum is observed for waves with a characteristic value of the wavenumber β^* in the lateral direction. In the low-frequency range, the value of β^* significantly exceeds the streamwise wavenumber α, which is characteristic of experimentally observed streamwise structures.

3. For stationary disturbances and M $= 0$, under changes in Re and $\beta =$ const, the highest efficiency of disturbance generation in the boundary layer corresponds to the value $(\beta\mathrm{Re})_1 \approx 0.7$, and under changes in β and Re = const corresponds to $(\beta\mathrm{Re})_2 \approx 0.55$. The latter value is in good agreement with the period of streamwise structures obtained in the experiments of [11].

4. The efficiency of disturbance generation in a supersonic boundary layer is lower than that in a subsonic boundary layer.

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